

Constructing a GDP-based Index for Use as a Benchmark

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Abstract

The gross domestic product (GDP) is a fundamental economic indicator that is frequently used as a benchmark for local equity indices. The widespread appeal of this association is understandable because an equity index, especially if broad, could, like the GDP, also manifest the state of the economy. At the same time, however, the validity of a ~~direct relation between the two is debatable since the GDP is known~~ to be characteristically different from the typical equity index, however broad.

In this work, we review some of the key elements that separate the GDP from a typical broad equity index in order to explain why the two cannot be compared directly with each other. We then incorporate a readily available mapping technique to create a GDP-based index that circumvents their inherent disparities and, thus, enable us to benchmark one against the other.

Keywords

GDP, benchmark, equity index, equity duration, relative valuation

1 Introduction

The GDP, in level or growth rate, is one of the most commonly used indicators that reflect the state of a country's economy. There is no need to describe here how this parameter is defined or estimated, as there is an extensive literature that covers it.

¹. The author expresses these views as an individual, not as a representative of companies with which he is connected.

What is relevant to this work is that this measure which, in one form or another, is sometimes used to benchmark local equity indices, begins first as a forecast over some time horizon, which is soon followed by a revision. The closed-loop revising process continues iteratively until the target date is reached, by which time convergence occurs. At this stage, the GDP becomes realized and, thus, “historical” and the process repeats itself with a new prediction towards a new target date. The time span for this cycle, from prediction to convergence, normally equals one quarter, which is the reporting time scale for many economic indicators.

The drive to speculate on what the future holds for the GDP,² as well as many other economic indicators, is fuelled by society’s need to invest and grow. A correct guess could guide one to the right investment decision which would, in turn, lead to higher productivity, growth in real profits and, subsequently, personal wealth. In fact, this speculative approach to linking the GDP with profits remains essential to the development of any competitive economy that thrives on investment and growth. At the other extreme, after the speculations have been revised over and over and the true GDP number comes to light, it will, purely by construct, embody the profits that underlie its constituents.

The circular relationship that the GDP has with profits, as summarised above, is therefore one reason why the indicator is used regularly as a benchmark for “broad”³ equity indices (Faugère and Van Erlach, 2006; Honnerová, 2003; and many others). Examples of such comparisons abound, whereby one simply has to open a financial publication (newspaper, magazine, etc.) to find them.

Considering all this, however, there is still the issue that a direct relationship between the two is not universally recognized (Kerschner et al. 1999). There are several reasons for this, among which are (1) the GDP is based mainly on revenues, whereas equity valuation, the very basis of any equity index, centers on profits; (2) a typical equity index has a higher exposure to foreign earnings than does the GDP; (3) the GDP level, or index, is measured in units of income (i.e. \$/year), whereas an equity index is denominated in value [i.e. \$]; and last, but not least, (4) by the time the GDP converges from speculation to historical, which could take up to several months depending on the forecast horizon, the number becomes actual. This contrasts sharply to what an equity index symbolizes, which is merely a speculation on the underlying firms’ earnings potential going forward.

On a macro level, one could, more or less, get around statements (1) and (2) above, but not (3) and (4). In reference to (1), for instance, one could claim that a company’s expenses comprise another company’s profits, so that a merger between the two can create a conglomeration that generates only revenues. Thus, while taking the two companies separately is akin to an equity index whose overall value is determinable by individual profits, their merger becomes analogous to the GDP, which tracks revenues.

2. In fact, there are publications that specialize in consensus forecasting, with the objective of narrowing down the views of economists and analysts to bring them into focus.

3. By broad, we mean an equity index that comprises a large number of public companies, belonging to a broad range of sectors and specializing in a broad range of products. Thus, the broader the index, the closer it comes to representing the economy as a whole.

Similarly, one could argue against (2) in that just as companies compete against each other economically, jurisdictions do so as well, but on a grander scale. For example, while an equity index in Country A, with an exposure to Country B, would contribute to, or gain from, the GDP of Country B, an equity index in Country B, with an exposure to Country A, would contribute to, or gain from, Country A's GDP. This closed cycle, therefore, would have a compensating effect on an equity index, enabling its foreign exposure to circle back into the GDP and, subsequently, the local equity indices and vice versa. As result, one should effectively be able to compare, albeit not directly, the characteristics of a country's GDP to the local, broad indices.

Statements (3) and (4), on the other hand, hinge on deeper fundamental discrepancies between the GDP level and an equity index and are therefore more challenging to crack. Here, as a consequence, one must follow a different path when trying to correlate the two parameters. Establishing this path and providing evidence for it, with intent to develop a GDP-based index for benchmarking purposes, constitute the remainder of this paper.

2 Relating the GDP with an Equity Index

As mentioned above, there are two primary reasons⁴ why the GDP cannot serve directly as a benchmark for an equity index. Firstly, different units of measure characterize the two parameters and, secondly, while the GDP reflects an actual, quantifiable number, the value of an equity index is entirely speculative. Proof for the latter can be observed in Figure 1, where the historical level of the US nominal⁵ GDP is displayed alongside the *S&P500*⁶ index, both plotted from 1975 to 2008. Here, although there is similarity in long-term trends, one could easily spot the difference in volatility between the two quantities, where the higher volatility in the *S&P500* is a manifestation of the speculations that shape it. In consequence, one must rely on an alternative way to accomplish the task of benchmarking an equity index, a speculative-based measure, against the GDP, which is a realized number, not to mention the other differences already listed above.

The alternative being considered here is founded on a readily existing methodology that can be also used to assess the relative valuation between the nominal level in *GDP* and an equity index. As a detailed derivation of the model is available elsewhere (Cohen, 2005), we shall avoid re-deriving it here and, instead, provide a brief summary of the notions that underlie it and the final outcome itself.

Beginning with the supposition that the value of a publicly traded security at a macro level (i.e. a broad equity index) depends on two parameters, one being the nominal⁷ interest rate and the other time⁸, it can be shown that

$$\ln S - bt = \Psi(b) \tag{1}$$

4. As given by statements (3) and (4) in the previous section.

5. Nominal instead of real because the equity index is in nominal terms.

6. Although the *DJIA* index is highly correlated with the *S&P500* and could have been used here to illustrate our example equally well, we have chosen to work with the latter primarily because it is broader and also used more popularly as a benchmark in a large number of studies.

7. See footnote 5.

8. Refer to Cohen (2005) for a justification of how the two parameters were selected.

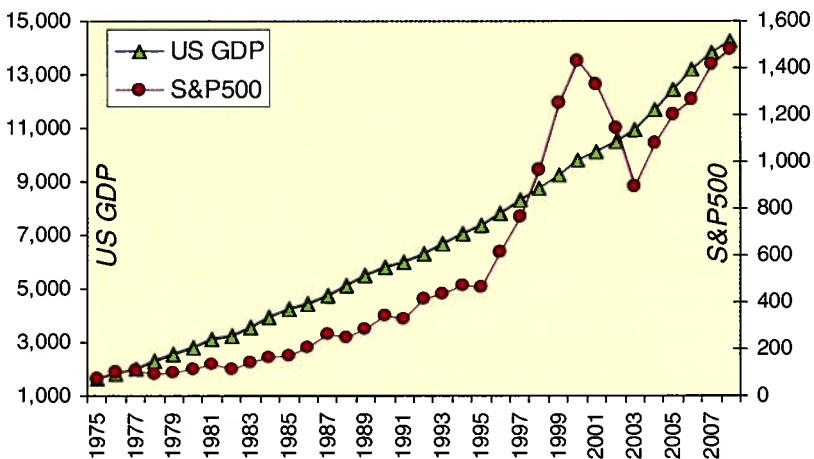


Figure 1: Comparing the trends of the nominal US GDP level and S&P500 index between 1975 and 2008. The higher volatility in the latter is a manifestation of its underlying speculative nature.

where S is the level of the index, b is the interest rate, t is an annual⁹ measure of time and $\Psi(b)$ is some function of b . There are several implications to the above relationship, more notably that Ψ turns out to be a function of *only* b . This means that a plot of Ψ against b within some time range t_1 to t_2 should, in the absence of any outliers and structural shifts in the economy, yield a *single*, continuous curve, depending only on b .

Table 1 provides an example of how Ψ is calculated from Equation 1, given S and b_{10} (b_{10} is the 10-year US government bond rate) at any time, t . In the case of 1977, for instance, where t , b_{10} and S are 2, 7.21% and 104, respectively, Ψ becomes $\ln(104) - 7.71\% \times 2 = 4.50$.¹⁰

The sample data in Table 1, starting with 1975, show a time parameter, t , which begins at 0, thus defining 1975 as the “reference” date. In fact, it does not matter what date is used as the reference, since the impact of this on the function Ψ is linear and cancels out in subsequent calculations.

Table 1 contains three additional elements, all which are manifested in Columns 6-8 and pertain to the *GDP*. The parameter G in Column 6 is the nominal level of the US GDP in billions of dollars. The treatment of G , also described in detail in Cohen (2005), leads to the following relationship

$$\ln V_G - bt = \Phi(b) \tag{2}$$

which is directly analogous to Equation 1. Here, however, V_G , which replaces the index, S , in Equation 1 represents the “value” of the economy at time t and is defined

9. The measure of time, t , is annual because the rate of interest is annual.

10. The reader may refer to Cohen (2005) for more examples and charts.

Table 1: An example calculation of the function Ψ in Column 5, given t , b and S in Columns 2-4, respectively. Columns 6-8, refer to the GDP, the treatment of which shall be discussed shortly

Year (1)	t (2)	B10 (3)	S&P500 (4)	Ψ (5)	G (6)	VG (7)	Φ (8)
1975	0	7.50%	73	4.28		24,337	10.10
1976	1	7.74%	97	4.50	1,825	26,239	10.10
1977	2	7.21%	104	4.50	2,031	31,827	10.22
1978	3	7.96%	90	4.26	2,295	32,202	10.14
1979	4	9.10%	100	4.24	2,563	30,654	9.97
1980	5	10.80%	111	4.17	2,790	28,967	9.73
					3,128		

(1) Date of measurement.

(2) Time span, t , using 1975 as the reference.

(3) The 10-year US government bond rate. (Data obtained from <http://www.federalreserve.gov/releases/h15/data.htm>)

(4) The S&P500 price index level. Source: <http://www.econ.yale.edu/shiller/data.htm>

(5) Ψ calculated using Equation 1.

(6) The nominal level of the US GDP. Source: <http://www.federalreserve.gov/releases/h15/data.htm> or <http://www.bea.gov/national/nipaweb/index.asp>

(7) Value underlying the economy (as defined in Equation 3).

(8) Φ computed using Equation 2.

as follows:

$$V_G(t) \equiv \frac{G(t+1)}{b(t)} \quad (3)$$

where $G(t+1)$ is the nominal GDP level one year ahead of t and $b(t)$ is the interest rate at time t . Therefore, since G is defined by units of earnings, i.e. \$/year, V_G will then acquire the units of value, i.e. \$, thus allowing direct association between Equations 1 and 3, representing an equity index and the GDP's implied value, respectively.

The reason for the 1-year gap in measurement time between G and b in Equation 3 has to do with how asset valuation is generally conducted in practice, which is based on forward looking earnings. In addition, if t represents today, $t+1$ would be one year from today, thus deeming $G(t+1)$ a forecast and, hence, a speculation, just like an equity index (refer again to Cohen (2005) for a more detailed explanation).

An example of how Φ is calculated follows from the sample data in Table 1. In reference to 1977, similar to the earlier illustrative calculation for Ψ , V_G is computed as $G(t+1)/b(t) = 2,295/7.21\% = 31,827$ leading, subsequently, to $\Phi(b) = \ln V_G - bt = \ln(31,827) - 7.21\% \times 2 = 10.22$.

3 Impact of Different Maturities

Although the interest rate, b , is a primary input to the model described above, its maturity has not been specified simply because it turns out to be irrelevant. What

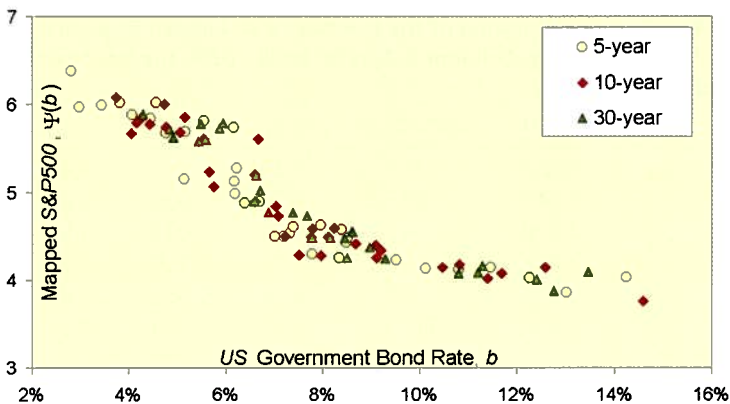


Figure 2a: The function $\Psi(b)$, defined by Equation 1 and based on the S&P500 index, is plotted here against b for different maturities. It illustrates, firstly, the irrelevance of the maturity of b and, secondly, the convergence of the data around b .

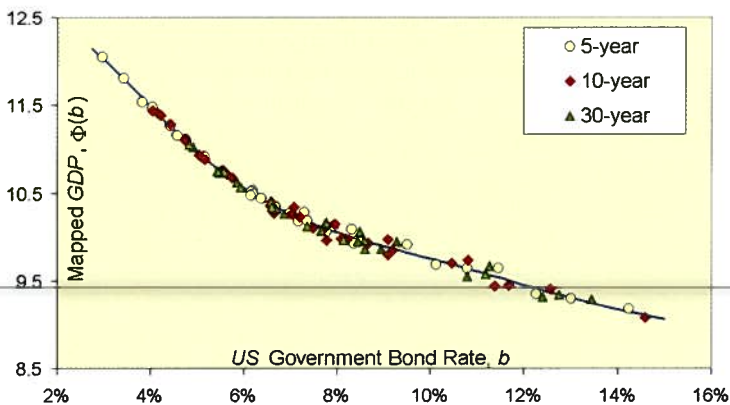


Figure 2b: The function $\Phi(b)$, defined by Equation 3, is plotted here against b for different maturities. Again, it depicts the irrelevance of the maturity of b , as well as the convergence of the data around b . The convergence is significantly tighter in this than in Figure 2a, owing to the fact that the GDP is a realized number rather than speculated. The line is a best-fit polynomial through all the points.

matters, though, is that b must be represented by a government bond rate, since a government bond is typically free of any firm-specific risks (credit, liquidity, etc.).

Figures 2a and 2b show Ψ and Φ , which relate to the S&P500 index and the US nominal GDP respectively, plotted against b for various US government bond maturities, namely 5, 10 and 30 years. The data range from 1975 to 2008, inclusive.

An important feature in Figures 2a and 2b is the convergence of both functions Ψ and Φ around b , which is implicit in how Equations 1 and 3 are derived. Nevertheless, Φ 's convergence is markedly tighter because the GDP is a realized number, whereas the equity index, which underlies Ψ in Figure 2a, is purely speculative. Another

prominent attribute, particularly in Figure 2b, is that the impact of the different rate maturities is indeed irrelevant.¹¹

Finally, for estimation purposes, the mapped GDP data in Figure 2b have been curve fitted with a polynomial of order 6, characterizing Φ as a function of b alone.¹² The single curve traverses all the different maturities considered, having taken into account their irrelevance.

4 Defining the New GDP-based Benchmark

With the above as background, we now proceed to construct the proposed GDP-based index for benchmarking purposes. For this, we refer to Figures 2a and 2b and note the difference in the levels across Ψ and Φ , which is exclusively a result of the disparity in the measurement scales between the GDP and the equity index, here being the S&P500. Subtracting a constant, $\alpha_{\Phi\Psi}$, from the level of Φ , thus shifting it down in parallel to coincide with Ψ , could easily circumvent this—i.e.:

$$\Psi(b) = \Phi(b) - \alpha_{\Phi\Psi} \quad (4)$$

which leads to

$$\ln S = \ln V_G - \alpha_{\Phi\Psi} \quad (5)$$

upon combining Equations 1, 2 and 4. Minimizing the sum of squared errors between the data points in Figure 2a and the fitted curve in Figure 2b gives $\alpha_{\Phi\Psi} \approx 5.41$.

Figure 3 depicts the fit between the curve in Figure 2b, as it is lowered by a constant value of 5.41 to the level of the data in Figure 2a. The comparison is satisfactory, attesting to a more direct relationship between the S&P500 index and V_G , as presented in Equation 5.

The constant, $\alpha_{\Phi\Psi}$, will, of course, vary with other indices, depending on their scales relative to V_G . As an example, $\alpha_{\Phi\Psi}$ for the *DJIA* index turns out to be approximately 3.30, whose fit with the same line in Figure 2b, but parallel shifted, is portrayed in Figure 4. The strong similarity between Figures 3 and 4, relating to the S&P500 and *DJIA* indices respectively, is due to the high correlation between the two indices. As high correlation among broad equity indices is generally the norm, the newly-constructed GDP-based benchmark, corrected for $\alpha_{\Phi\Psi}$, could thus have large-scale applications.

Next, upon exponentiating Equation 5, one can finally define S as a GDP-based benchmark for any broad equity index. In the case of S&P500, for instance, incorporating 5.41 for $\alpha_{\Phi\Psi}$ leads to:

$$S_{S \& P500} = \exp(-\alpha_{\Phi\Psi}) \times V_G = V_G/223.6 \quad (6)$$

11. More examples of this nature, related to different jurisdictions, can be found in Cohen (2005).

12. The 6th-order best-fit polynomial is given by $\Phi(b) = -9700930 \times b^6 + 5819200 \times b^5 - 1375820 \times b^4 + 160458 \times b^3 - 9257.7 \times b^2 + 199.4 \times b + 11.0$ with an R^2 of 99.4%. The use of this choice of curve is driven primarily by the high R^2 it can achieve.

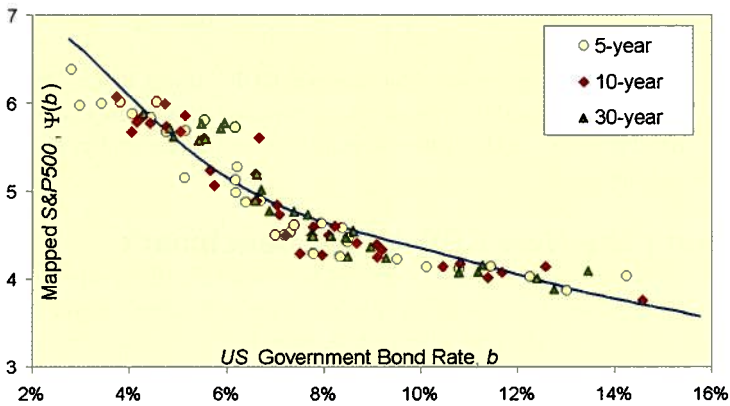


Figure 3: Lowering the best-fit curve in Figure 2b by a constant, $\alpha_{\phi\Psi}$, equal to 5.41 and corresponding to the S&P500 index, and inserting into Figure 2a to compare with $\Psi(b)$.

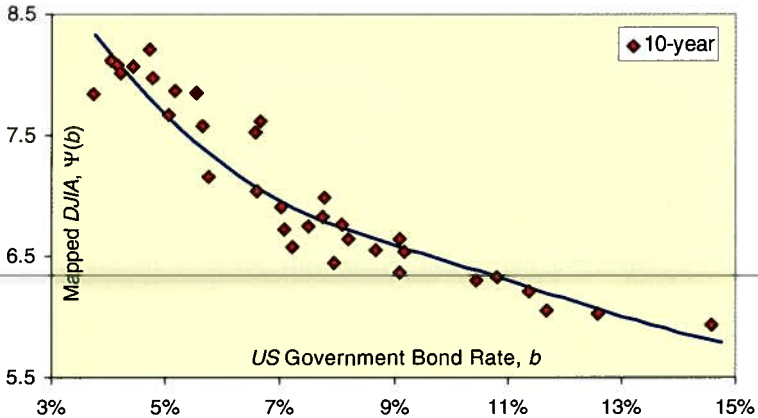


Figure 4: Similar to Figure 3 and for the same time range, but for the DJIA index. The solid line is the best-fit line in Figure 2b, but with a parallel shift and an $\alpha_{\phi\Psi}$ of about 3.30. To reduce the clutter, the figure includes the function Ψ based on only b_{10} .

where $S_S \& P_{500}$ represents the benchmark for the S&P500 index. Figures 5a-b compare the S&P500 index against its GDP-based benchmark, $S_S \& P_{500}$, both plotted against the year, ranging from 1975 to 2008. Being linear in scale, Figure 5a amplifies the differences at higher levels, whereas its logarithmic counterpart, Figure 5b, helps provide an objective comparison over the entire range of the time scale.

Prior to continuing, we observe that the benchmark in Figures 5a-b is based on b_{10} only, knowing well that V_C in Equation 6 depends on the maturity that one ultimately chooses. The reason for focusing on a single rate, rather than all three considered here, is essentially to minimize the clutter in the graph and produce a clearer picture.

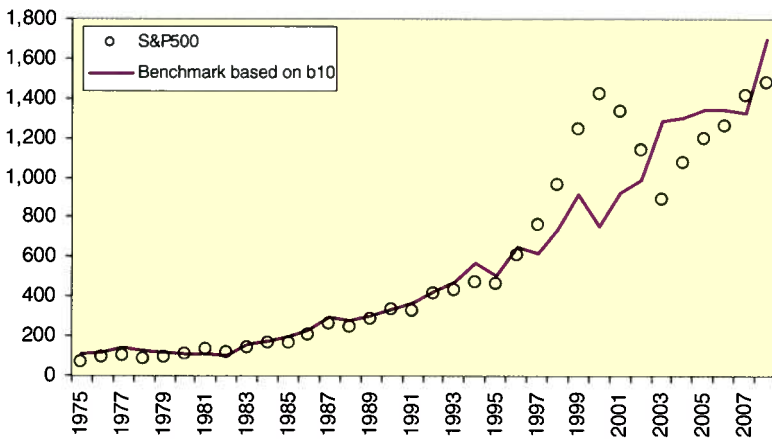


Figure 5a: Comparison of the new GDP benchmark, as calculated from Equation 6 and based on b_{10} , with the S&P500 index.

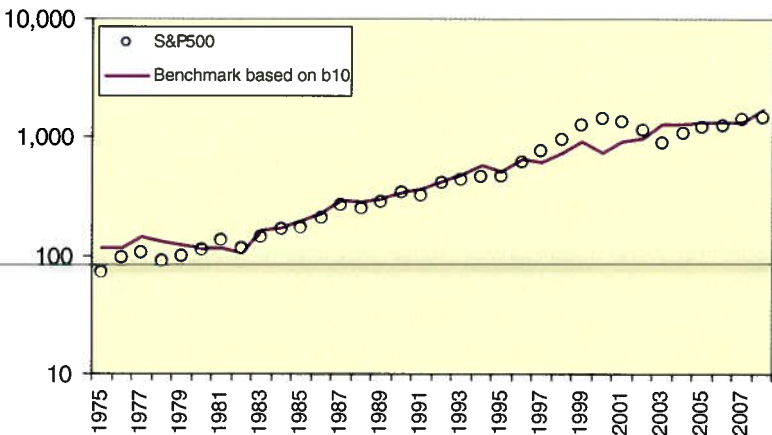


Figure 5b: Same as Figure 4a, but in logarithmic scale.

Figure 6, on the other hand, is included here to depict how the proposed GDP benchmark might vary depending on the different interest rate maturities that could go into calculating V_G . In this example, where the behavior of the benchmarks is displayed based on the 5- and 10-year maturities, we observe that, although the long-term trend is very similar in both cases, there appear to be gaps that separate them in certain time regimes. In this respect, it is worth mentioning that wider gaps in Figure 6 are caused by larger spreads in the corresponding yield curves, bearing in mind that this is perhaps an outcome of underlying economic uncertainties. In contrast, tighter gaps result from more horizontal yield curves, which, as discussed in Cohen (2006), could manifest periods of higher economic certainty. Therefore, as described here, the link between the maturity-induced gaps

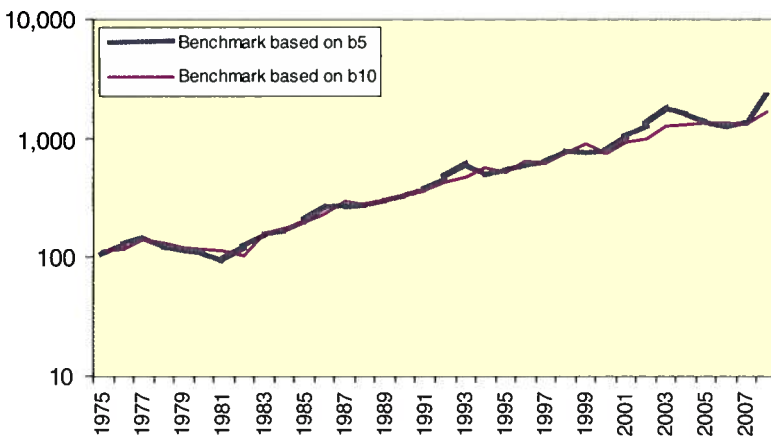


Figure 6: Illustration of how the GDP-based benchmark, as given by Equation 6, differs depending on the maturity used to calculate V_G . The gaps between the benchmarks reflect the spread in the underlying yield curve, whereby larger spreads will lead to wider gaps and vice versa.

in the benchmarks and the spreads in the underlying yield curves could, potentially, explain how economic uncertainty is transmitted to equity markets.

We finally refer back to Figures 5a-b and draw two important conclusions. Firstly, the issue of different measurement units, as discussed earlier and depicted by the use of two separate axes in Figure 1, is now resolved. The proposed method effectively places both the equity index and its benchmark into the same category of units, thereby allowing direct comparison between the two. Secondly, there is evidence of a better fit between the S&P500 index and its benchmark in Figures 5a-b than that in Figure 1. In essence, the index in Figures 5a-b appears to follow its GDP-based benchmark more closely than it does the GDP alone. The gaps in Figures 5a-b are, in addition, observed to occur in times well known for their characteristic market slumps (i.e. mid 1970s and between 2001-2004) and bubbles (1997–2000). Clearly, therefore, the benchmark is also capturing the relative valuation effect that is embedded in its derivation.

The appeal of the new benchmark goes beyond its satisfactory fit with the equity index it allegedly portrays, or its ability to highlight periods of over/under valuation of the index. Since the benchmark can be described in closed form, one should then also be able to estimate its duration—an important property—more easily and objectively, as it shall be demonstrated next.

5 Calculating the Duration of the New Benchmark

The duration of a financial instrument is normally defined as its sensitivity to the interest rate, all else constant. There are certain issues with this generic definition, however, which introduce a hurdle when it comes to practical implementation. These issues are two-fold and provoked mainly by the following questions: (1) sensitivity to what interest rate in the yield curve and (2) how does one get around “all else constant”, which is critical to the definition? Although these are non-issues

when it comes to determining the duration of a bond, owing to the existence of a close-form valuation relationship, they are major when equities, equity indices and their related benchmarks are involved. This is likely due to the lack of an objective and closed-form equity valuation relationship, as well as the absence of some measure of maturity or investment horizon.

The literature, however, does contain a number of works related to computing the duration of equity. Here, for instance, there is the notion that a combination of the book-to-market value and other ratios can provide a proxy for the (implied) duration (Dechow et al. (2004) and Santa-Clara (2004)). In the case of equity portfolios and, likewise, indices, the use of the Dividend Discount Model [DDM] for estimating duration appears to dominate, although different authors have offered tweaked versions of it. For example, while Casabona et al. (1984) suggest directly implementing the DDM, Leibowitz et al. (1989) propose incorporating a discount rate that takes into account an equity risk premium sensitive to both inflation and real rates. As related works are abundant in the literature, it is perhaps better, in the interest of space, not to delve deeply into them and, instead, proceed directly with obtaining the duration of the new GDP-based benchmark introduced here.

To get this, we combine Equations 2 and 5 and arrive at:

$$\ln S = \Phi(b) + bt - \alpha_{\phi\psi} \quad (7)$$

where S denotes the benchmark, as defined earlier, and $\alpha_{\phi\psi}$ a constant. Now, based on definition, we write the following

$$D_S \equiv - \left(\frac{\partial \ln S}{\partial b} \right)_t \quad (8)$$

with D_S representing duration, $\partial \ln S / \partial b$ denoting sensitivity to the interest rate and the subscript t symbolizing “all else constant”. Substituting 7 into 8 finally yields:

$$D_S = - \left[\frac{d\Phi(b)}{db} + t \right] \quad (9)$$

Given the above, along with an estimate for $\Phi(b)$ based on the curve fit in Figure 2b (see polynomial in Footnote 12), one could subsequently compute the duration of the benchmark, D_S , without difficulty and in closed form. The choice of b — be it b_5 , b_{10} , b_{30} , or any other that is preferred—is also easily implementable, owing to the irrelevance property discussed earlier. How D_S varies with the choice of b , therefore, would be indicative of the benchmark’s sensitivity to the different maturities.

A sample calculation of the duration’s measure, as presented by Equation 9, is shown in Table 2.

The duration calculation in Table 2 has been extended to include the rates b_5 , b_{10} and b_{30} , as well as taken through to 2008, with the outcome plotted in Figure 7. Here, however, we have focused on 1990 onwards, so as to once again reduce clutter.

Figure 7 contains some interesting features. Firstly, the irrelevance principle discussed earlier seems to apply to duration as well, as the durations computed for the different rate maturities considered here appear to more or less cluster uniformly around a common trend. This independence of maturity means, for

Table 2: Sample calculation of duration in accordance with Equation 9. A 10-year rate has been selected here as the example

Year (1)	t (2)	b_{10} (3)	Φ (4)	Φ' (4)	G (6)
1975	0	7.50%	10.14	-20.7	20.7
1976	1	7.74%	10.10	-19.2	18.2
1977	2	7.21%	10.21	-23.0	21.0
1978	3	7.96%	10.06	-17.9	14.9
1979	4	9.10%	9.83	-14.3	10.3
1980	5	10.80%	9.63	-14.8	9.8
1981	6	12.57%	9.37	-14.9	8.9

(1) Date of measurement.

(2) Time span, t , using 1975 as the reference.

(3) The 10-year US government bond rate. (Data obtained from <http://www.federalreserve.gov/releases/h15/data.htm>)

(4) Estimated $\Phi(b)$ based on the polynomial curve fit in Figure 2b. See Footnote 12.

(5) Derivative of Φ with respect to b , as calculated from the polynomial curve.

(6) Duration calculation based on Equation 9.

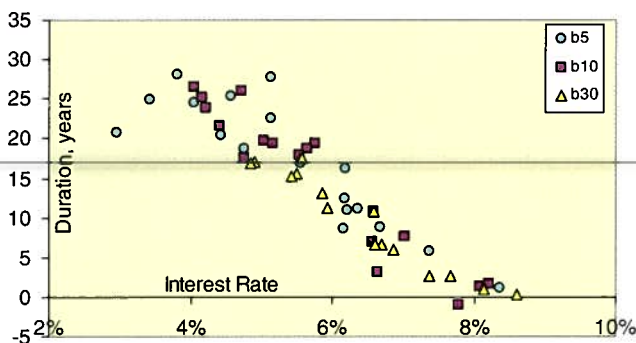


Figure 7: Plot of the duration of the proposed benchmark, in accordance with the calculation procedure outlined in Table 7. Duration here is shown for different rates, namely b_5 , b_{10} and b_{30} , spanning from 1990 to 2008.

instance, that the duration of the benchmark relative to b_5 would be equal to that relative to b_{10} , if the two rates were to be the same at a given time, t . This is not surprising because the fundamental input, Φ , into calculating the benchmark's duration (see Equation 9) is independent of maturity. Secondly, there seems to be a clear trend that increases with decreasing interest rates, implying that the overall fall in interest rates, which has been observed since 1990, has led to a more rate-sensitive benchmark. Since the benchmark's long-term trend is a reflection of the major equity indices (i.e. see Figure 5b, for example, for the S&P500), one could

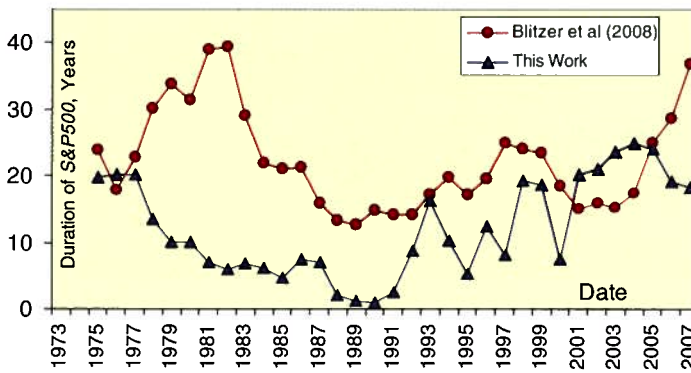


Figure 8: Comparison of the duration of the proposed benchmark, as calculated from Equation 9, with that provided in Blitzer et al. (2008).

conclude that indices have also, on the whole, experienced increased sensitivity to interest rates since 1990.

To compare with other works, we display in Figure 8 the duration of the above-mentioned benchmark plotted alongside that of the S&P500 index, the latter estimated using the method outlined in Blitzer et al. (2008). The triangles, which summarize the outcome of this work, depict the duration values averaged over the three maturities considered herein, namely b_5 , b_{10} and b_{30} .

Prior to comparing the two, we should note that, firstly, the duration estimated in Blitzer et al. (2008) relates to S&P500, which has been readily benchmarked here against the proposed index, with results shown in Figures 5a-b. Secondly, the method underlying the reference is based on the DDM, which implements certain assumptions on the risk premium, growth, corporate rating and maturity and, furthermore, incorporates moving averages and the like.

That said, the two lines in Figure 8 display certain disparities, as well as similarities. For instance, the estimate of Blitzer et al. (2008) depicts a rising trend in duration during 1977–1981 and 2005–2007, whereas this work shows it to be falling. With the exception of these relatively short periods, the long-term trends appear to be more or less in line with each other. This goes along with the observation that none of the curves crosses zero at any point in time.¹³ As for their estimated magnitudes, the duration measures do seem to correspond better with each other post rather than prior 1992. Altogether, our work shows the GDP-based benchmark to possess a higher volatility, perhaps due to the absence of any averaging scheme, moving and otherwise, within it.

We now outline another approach to estimating the duration of the newly proposed benchmark. Returning to Equation 7 and taking its total differential with

13. One of the points in Figure 7 is clearly below the zero line, although it does not show up in the triangles in Figure 8. The reason for this is that the durations in Figure 8 are averaged over the three rate maturities, thus washing out this statistical anomaly. It should be pointed out that no similar anomalies were observed in relation to any of the other data points throughout the entire time frame 1975–2008.

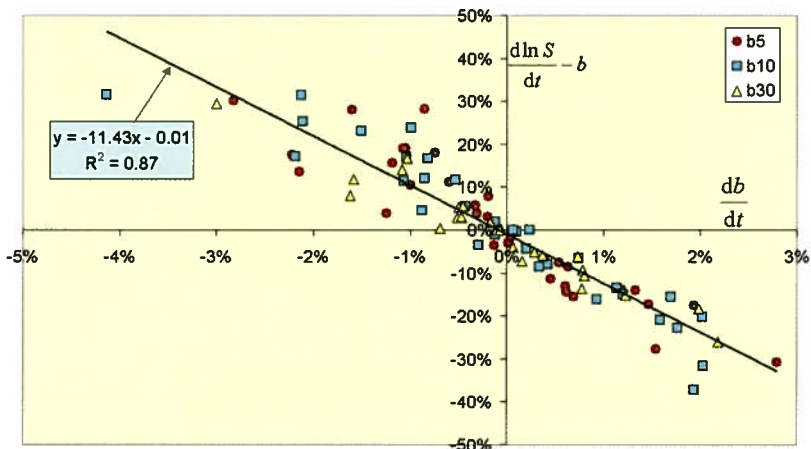


Figure 9: Plot of $d \ln S / dt - b$ vs db / dt over the time period 1975–2008 and for the three maturities considered. According to Equation 10, the slope of the best-fit straight line should represent the average duration, here being about 11 years.

respect to time, t , leads to:

$$\frac{d \ln S}{dt} - b = -D_s \frac{db}{dt} \quad (10)$$

upon combining with Equation 9 and rearranging. Equation 10 simply states that a plot of the left-hand side of Equation 10 vs the time rate of change in interest rate, db / dt , all which can be assessed from available data (such as in Table 1), should provide information on the value of D_s . Figure 9 represents exactly this, where $\frac{d \ln S}{dt} - b$ is plotted against db / dt over the time period 1975–2008 and for the three maturities considered.¹⁴ Without going into any depth, it is noted that a best-fit straight line passing through all the points has a slope of -11.4 , suggesting that the average duration of the benchmark over the date range and maturities is about 11 years. This value is in good agreement with the overall average exhibited by the triangles in Figure 8. Equally important here is the absence of a statistically significant intercept, which is in full accord with the theoretical form of Equation 10.

Finally, we include Figure 10, which is similar to Figure 9 but, for sake of clarity, focuses on a single maturity and a narrower time frame, namely b_{10} and 1990–2008, respectively. The average duration over this time period is estimated at roughly 16 years, somewhat larger than the overall average value indicated earlier. The reason for this is the rising trend in duration after 1990, as observed in Figure 8, and also consistent with the data in Figure 7, which portray an increase in duration with falling interest rates.

14. The time derivative, d / dt , is estimated here in annual difference, so that, in reference to Table 1, db / dt corresponding to 1976, i.e. $t = 1$, would be approximately the difference between $b(1976)$ and $b(1975)$, or $7.74\% - 7.50\% = 0.24\%$.

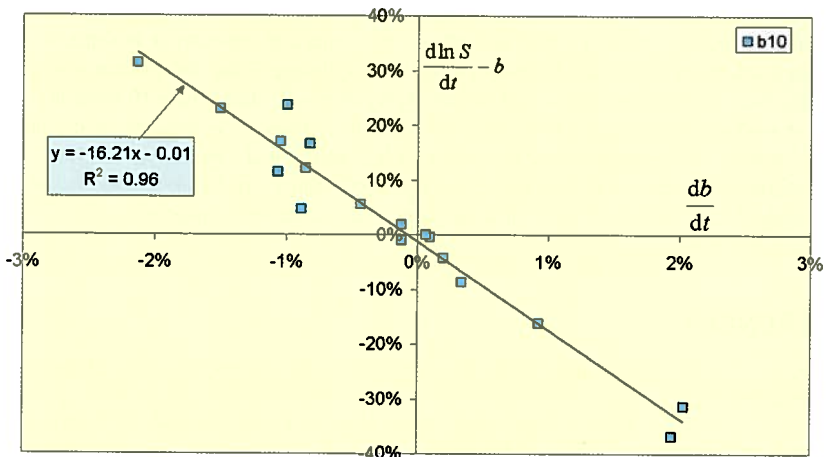


Figure 10: Same as Figure 9, but focusing only on b_{10} and the time frame 1990–2008. The average duration for b_{10} throughout this time period is shown to be about 16 years. Also, the absence of a statistically significant intercept is consistent with the form of Equation 10.

6 Conclusion

This work addresses some of the issues surrounding the direct implementation of the GDP as a benchmark for broad equity indices and suggests a way for getting around them. It is shown here that representing the GDP by its underlying value, rather than incorporating it on a standalone basis, produces not only a better fit as a benchmark, but also has other beneficial uses, such as providing a measure of relative valuation, whereby one could identify periods of under or over valuation of the index against which the benchmark is used.

Following on, the benchmark's duration is investigated as well, offering two distinctive and objective ways for measuring it, both of which seem to generate consistent results. While one approach establishes the trend, the other concentrates on estimating the average over a given time period, leading to the conclusion that the duration of the US GDP-based benchmark relative to the US government rates averages at about 11 years over the time period 1975–2008, but increases to 16 years over the period covering 1990–2008. Moreover, the benchmark's duration appears to have risen since 1990 as interest rates have fallen gradually, a trend that seems to relate as well to the duration of the S&P500.

Finally, we reiterate that this work touches only the surface of this very important area and thus leaves many questions unanswered. For instance, are the relationships and conclusions derived here universal and applicable equally across borders and over longer time horizons? Also, could certain situations, where a connection between the GDP-based benchmark and the underlying equity index is undoubtedly absent, point to underlying data issues or even the possibility of hidden market manipulations and inefficiencies? An extension of this work could potentially help address these questions and, perhaps, many others.

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REFERENCES

- Blitzer, D.M., Dash, S. and Murphy, P. 2008. Equity Duration—Updated Duration of the S&P500. Standard & Poor's.
- Casabona, P., Fabozzi, F. J. and Francis, J. C. 1984. How to Apply Duration to Equity Analysis. *The Journal of Portfolio Management* **10**: 52–54.
- Cohen, R.D. 2005. The Relative Valuation of an Equity Price Index. In: Paul Wilmott (ed.) *The Best of Wilmott 2*, John Wiley & Sons, Ltd, 99–132.
- Cohen, R.D. 2006. A Var-based Model for the Yield Curve. *Wilmott Magazine*, May: 60–68.
- Dechow, P.M, Sloan, R.G. and Soliman, M.T. 2004. Implied Equity Duration: A New Measure of Equity Risk. *Review of Accounting Studies* **9**: 197–228.
- Faugère, C. and Van Erlach, J. 2006. The Equity Premium: Consistent with GDP Growth and Portfolio Insurance. *The Financial Review* **41**: 547–564.
- Honnerová, J. 2003. Time Series Analysis of GDP and Market Indices. *Bulletin of the Czech Econometric Society* **10**: 35–64.
- Kerschner, E.M., Doerflinger, T.M. and Murphy, D.B. 1999. What is the S&P? PaineWebber Investment Policy Report.
- Leibowitz, M.L., Sorensen, E.H., Arnott, R.D. and Hanson, H.N. 1989. A Total Differential Approach to Equity Duration. *Financial Analysts Journal* **45**: 30–37.
- Santa-Clara, P. 2004. Discussion of 'Implied Equity Duration: A New Measure of Equity Risk'. *Review of Accounting Studies* **9**: 229–231.
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